

Exploring the interplay between AI and human logic in mathematical problem-solving

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Abstract

This paper investigates the dynamic interplay between Artificial Intelligence (AI) and human logic in the domain of mathematical problem-solving. By critically examining a series of case studies, we compare the efficacy of AI-generated solutions, particularly those offered by ChatGPT, against traditional human problem-solving methods. The study employs various mathematical challenges, ranging from abstract logical puzzles to applied numerical problems, to evaluate AI's problem-solving approach and alignment with human cognitive processes. Our analysis highlights instances where AI's computational strategies complement or diverge from human reasoning, shedding light on AI's potential and limitations in deciphering mathematical problems. Furthermore, we explore the implications of integrating AI tools in educational contexts, specifically, their role in enhancing students' mathematical problem-solving skills. The paper aims to contribute to the ongoing discourse on the optimal utilization of AI in education, proposing a balanced approach that leverages AI's computational power while fostering the depth and creativity of human logic. Through this comparative study, we advocate for a collaborative model where AI and human reasoning merge to enrich the educational landscape, particularly in the teaching and learning of mathematics.

Keywords: Artificial intelligence, human logic, mathematical problem solving, ChatGPT capabilities, educational technology, cognitive processes, learning and knowledge agents.

Introduction

Integrating Artificial Intelligence (AI) in educational systems has opened new frontiers in teaching and learning methodologies. With the advent of advanced AI models like ChatGPT, there is a growing interest in harnessing AI's potential to solve complex mathematical problems, a domain traditionally dominated by human cognitive abilities. This paper explores the intersection of AI and human logic in mathematical problem-solving, examining how AI algorithms approach problems compared to human reasoning. Several studies have compared solutions obtained by AI and human logic to various mathematical problems. One research provided examples of new fundamental results in pure mathematics that have been discovered with the assistance of machine learning—demonstrating a method by which machine learning can aid mathematicians in discovering new conjectures and theorems (Davies et al., 2021). Another study utilized prompt engineering and Chain of Thought to summarize general tips for given mathematical problems (one geometry problem and one algebra problem) and their solutions (Jia et al., 2024). A Turing test was conducted in this study to compare ChatGPT-generated general tips with human-designed ones, which showed that the average score for ChatGPT-generated tips is less than that of human-designed tips at a statistically significant level ($p < 0.05$). AI-generated general tips can be a valuable reference for teachers to enhance efficiency and students' mathematical learning (Jia et al., 2024).

Literature Review

The literature on AI's role in education suggests that AI can offer personalized learning experiences and adapt teaching methods to individual student needs (Laak & Aru, 2024). However, when it comes to solving mathematical problems, the ability of AI to mimic human logical processes remains a subject of debate. Turing's seminal work on machine intelligence laid the groundwork for exploring AI's capabilities in logical reasoning (Turing, 1950), while more recent studies have investigated AI's application in specific educational contexts, including mathematics (Mohamed et al., 2022).

Recent research by Gao et al. (2023; 2024) highlighted AI's challenges in tasks requiring deep understanding and creative problem-solving, where human reasoning excels. Conversely, the study's examination of AI in education underscores AI's potential to support learning by offering new problem-solving strategies. A recent study by Raisch and Fomina (2024) argued that the outcomes of human-AI collaboration are contingent on the processes employed to combine human and artificial intelligence. Their research further states that while routine tasks can be automated, exploratory tasks that require new problem-solving methods warrant a hybrid approach that integrates human intelligence with AI. Further, it suggested a model that incorporates a three-pronged hybrid problem-solving process and their respective outcomes: autonomous search, which generates more distant solutions; sequential search, which enables more local solutions; and interactive search, which promotes more re-combinative ones. These three approaches provide a wide range of options for organizations when searching for outcomes for problem-solving. Davies et al. (2021) suggested a model for collaboration between the fields of mathematics and AI. Their study demonstrated a framework for using machine learning to discover potential patterns and relations between algebraic and geometric structures of knots and a proposed resolution to a long-

standing open conjecture in representation theory. This framework's success allows mathematicians to understand better the behavior of objects that are too large for them to observe patterns otherwise. Davies et al. (2021) study furthers the introduction of machine learning into mathematicians' work and encourages further collaboration between the two fields. A systematic literature review of AI in mathematics education in ScienceDirect, Scopus, Springer Link, ProQuest, and EBSCO Host for 20 AI studies published between 2017 and 2021 showed that the AI approach used in mathematics education for the samples studied were robotics, systems, tools, teachable agents, autonomous agents, and a comprehensive approach. The analysis revealed that using AI during teaching and learning will make learning more interactive. Additionally, there are several advantages to using AI in mathematics learning, among which are that students become more critical and responsible in facing daily solutions and better understand fundamental problems of geometry, mathematics, and statistics (Mohamed et al., 2022).

Computers outperform humans in solving mathematical puzzles involving both text and diagrams, as demonstrated in the research (Chesani et al., 2017). Moreover, they proposed solving mathematical puzzles using computers without human intervention, starting with text and diagrams and describing them. They suggested bringing insights into collaborative interactions between humans and machines that would also be beneficial for disseminating artificial intelligence in educational settings (Chesani et al., 2017). This paper contributes to this ongoing discourse by presenting a comparative analysis of AI-generated and human-derived solutions to various mathematical problems. Through this comparison, we aim to elucidate the nuances of AI's problem-solving approach, assess its alignment with human reasoning, and explore the implications of AI-assisted learning in educational settings.

Mathematical Problems and Solutions

Problem

When Alice was born, her mother was three times that of Alice's current age, and her mother was twice that of Alice's current age.

- (a) Alice, her mother, her father, who is the oldest, and who is the youngest? Why?
- (b) What is the average age of Alice, her mother, and her father? Why?

Assume that there is no error in this problem.

A Wrong Answer was Provided by ChatGPT4 (6:48 AM on March 23, 2024)

Alice is the youngest at 18 years old.

Alice's mother is next, currently stated to be 48 years old.

The father's age is not provided, but we can infer he might be older than the mother if traditional age dynamics apply, as the problem suggests a patriarchal structure by listing the father last. Without the father's age, we cannot calculate the average age of all three individuals. If there is additional context or specific ages for all involved, that would be crucial to solving the problem accurately.

The average age of Alice, her mother, and her father is unknown.

Another Wrong Answer was Provided by ChatGPT4 (6:52 AM on March 23, 2024)

The oldest is Alice's mother at 72 years old.

The youngest is Alice at 18 years old.

The average age of Alice and her mother is 45.

Solution Based on Logic and Reasoning

It is clearly stated that there is no error in the problem.

When Alice was born, her mother was three times that of Alice's current age, and her mother was twice that of Alice's current age. Let Alice be x years old now. Her mother was $3x$ years old when Alice was born. Her mother was $2x$ years old when Alice was born. Thus $3x=2x$. Therefore, $x=0$. It is not possible logically.

Therefore, there is no solution to this problem.

Problem

Let n be an integer. Find the median of $-n, 0, 4n, n, -4n, 1234 \times 0$.

A Wrong Answer was Provided by ChatGPT4 (7:44 AM on March 23, 2024)

For $n>0$, the median is 0.

For $n<0$, the median is $-(n/2)$.

For $n=0$, the median is 0

Correct answer

The median is 0.

If $n=0$, all the six numbers are the same. The median is 0.

If $n>0$, we arrange the six numbers in increasing order: $-4n, -n, 0, 0, n, 4n$. The median is 0.

If $n<0$, we arrange the six numbers in increasing order: $4n, n, 0, 0, -n, -4n$. The median is 0.

Problem

Alice is a 40-year-old lady. We do not know her biological mother or biological daughter's age. What is the average age range of Alice, her biological mother, and her biological daughter?

An Unreasonable Answer Provided by ChatGPT4 (8:31 AM on March 23, 2024)

Alice's Mother's Age: Assuming Alice's mother had her between the ages of 20-30, her mother could be approximately 60-70 years old now ($40 + 20$ to $40 + 30$). Alice's

Daughter's Age: If Alice had a daughter between the ages of 20-40, her daughter could be anywhere from a newborn to 20 years old. The estimated average age range for Alice, her

biological mother, and her biological daughter spans from approximately 33.3 years to 43.3 years.

A Mathematically and Logically Reasonable Answer

Erramatti Mangamma currently holds the record for being the oldest living mother who gave birth at the age of 73 through in-vitro fertilization via cesarean section in the city of Hyderabad, India (Wikipedia, 2024a).

Lina Marcela Medina de Jurado (Spanish pronunciation: ['lina me'dina]; born 23 September 1933) is a Peruvian woman who became the youngest confirmed mother in history when she gave birth on 14 May 1939, aged five years, seven months, and 21 days (Wikipedia, 2024b).

We can assume that Alice's mother is 45 years old now and Alice's daughter is 0 years old. $(45+40+0)/3 = 28.333$. If we assume that Alice's mother is now 113 years old and Alice's daughter is 35 years old. $(113+40+35)/3 = 62.667$. Therefore, the average age range is $[28.333, 62.667]$.

Problem

Alice has an annual income of one billion dollars and 77 cats, 88 donkeys, and one hundred and one dalmatians. When she was born, her mother was 100 times Alice's age. What is the average age of Alice and her mother? Assume ages are positive integers of years.

An Unreasonable Answer Provided by ChatGPT4 (9:48 AM on March 23, 2024)

Let A represent Alice's current age.

The average age of Alice and her mother.

This answer is logically incorrect, although it is a mathematical sound.

Logically Reasonable Answer

Let Alice be one year old. Her mother was 100 years old when Alice was born. That is not possible according to the reference in a previous problem. Let Alice be sold for at least two years. Her mother was at least 200 years old when Alice was born. That is not possible. Therefore, there is no solution to this problem.

Problem

We cannot expect a solution or correct answer from ChatGPT for many questions. For instance, what is your average age and that of your mother?

Response Provided by ChatGPT4

As an AI developed by OpenAI, I do not have a personal age, family, or experiences as humans do, so I cannot have a mother or an age of my own. My existence is based on algorithms, data, and programming designed to understand and generate human-like text based on the input I receive. If you have questions about calculating averages or any other inquiries, please ask!

Google Cloud Platform: Colab

Google Compute Engine delivers Virtual Machines (VMs) running in Google's innovative data centers and worldwide fiber network. Computing Engine's VMs boot quickly, come with persistent disk storage, and deliver high performance. Setting up Google Colab (n.d.) can be done by following the four steps:

1. Making sure the computer or other device (e.g. cellphone, Mac, PC, Chromebook, iPad, or Raspberry Pi) is connected to the internet.
2. Log in with a Gmail account using Google Chrome or another web browser.
3. Open Google Chrome or another web browser logged into to via the Gmail account.
4. In the Search URL bar located at the top of Google Chrome or another web browser, enter <https://colab.research.google.com/> and press enter.

Problem

Given the following information, please find the Tooth Fairy's age.

If her age is divided by 11, the remainder is 4.

If her age is divided by 9, the remainder is 3.

If her age is divided by 20, the remainder is 12.

If her age is divided by 25, the remainder is 7.

Her age is the smallest number between 10,000,000,000 and 1,000,000,000,000.

Response Provided by ChatGPT4 (10:57 AM on March 23, 2024)

We will use the divisors' least common multiple (LCM) (11, 9, 20, 25) to increment our search steps for efficiency. Let us proceed to find the smallest age that meets all the conditions within the specified range. We still face challenges in obtaining the result for the Tooth Fairy's age within this computational environment, even after adjusting the range. This task requires processing a large set of numbers with specific conditions, which might be too intensive for this environment. The method we have discussed is valid, but executing it might be more feasible in a more robust computational environment or specialized software designed for such tasks.

Solving the Problem Efficiently by Using Logic, Mathematics, Algorithms, and Python in Colab

The following is the Python program.

```
import time
start_time = time.time()
k= 4*10**8
for k in range(4*10**8,4*10**10+1):
    if ((25*k+7) % 11 == 4 ) and ((25*k+7) % 9 == 3 ) and ((25*k+7) % 20 == 12 ):
        print(25*k+7)
        break
    else:
        k = k+1
end_time = time.time()
print('Execution times is', end_time - start_time, 'seconds.')
```


It took 0.007 seconds to find the Tooth Fairy's age, 10,000,001,532.

Problem

Snow Pea Queen is Bean King's first princess. Do you know how old she is now? Let us find her age from the following information.

If her age is divided by 11, the remainder is 4.

If her age is divided by 9, the remainder is 3.

If her age is divided by 20, the remainder is 11.

If her age is divided by 25, the remainder is 7.

Her age is the smallest number between 1,000,000,000,000 and 1,000,000,000,000,000.

Response Provided by ChatGPT4 (11:28 AM on March 23, 2024)

Given the extensive range, we will use the divisors' least common multiple (LCM) (11, 9, 20, 25) to increment our checks for efficiency. Let us find the smallest age that satisfies all the given conditions within the specified range. Finding Snow Pea Queen's age within the given range and conditions is too computationally intensive for this environment, leading to an automatic interrupt due to the long processing time. Given the specific conditions and the extensive range, this task might require a more robust computational setup or specialized mathematical software.

A Solution Based on Logic and Mathematics

Let us investigate mathematics.

From $n \% 20 = 11$ and $n \% 25 = 7$, we know that n must be an odd number.

Let $n = 50k + 7$.

$(50k + 7) \% 20 = 11$

$(10k + 7) \% 20 = 11$

If k is even, $(10k + 7) \% 20 = 7$.

If k is odd, $(10k + 7) \% 20 = 17$.

Therefore, Snow Pea Queen's age does not exist. (That is, there is no solution to the problem.)

Problem

Say Tom is a good boy with a probability of 0.6 or a bad boy with a probability of 0.4. If Tom is a good boy, he gets a treat with a probability of 0.5, but if Tom is a bad boy, he gets a treat with a probability of 0.1. You see Kate giving Tom a treat. Based on your observation, what probability is that Tom was a good boy?

Solution provided by ChatGPT4 (8:21 PM, April 1, 2024)

Given that Tom received a treat, we use Bayes' theorem to find the probability he was a good boy. Using probabilities: $P(G) = 0.6$, $P(B) = 0.4$, $P(T|G) = 0.5$, and $P(T|B) = 0.1$, we compute $P(T) = 0.5 \times 0.6 + 0.1 \times 0.4 = 0.34$. Applying Bayes' theorem: $P(G|T) = \frac{P(T|G) \times P(G)}{P(T)} = \frac{0.5 \times 0.6}{0.34} \approx 0.882$. Thus, the probability that Tom was a good boy given he received a treat is approximately 88.2%.

Questions to ChatGPT4 and its responses (8:24 PM, April 1, 2024)

What can students learn if they use ChatGPT to get the above answer? What are the advantages and disadvantages?

Students using ChatGPT to obtain the above answer can learn several things:

Bayes' Theorem: They can learn about Bayes' theorem, a fundamental concept in probability theory, and see how it is applied to solve real-world problems.

Probability Calculations: They can learn how to calculate conditional probabilities using Bayes' theorem and the law of total probability.

Problem-Solving Skills: They can develop problem-solving skills by breaking down complex problems into manageable parts and applying appropriate mathematical methods to find solutions.

Advantages of using ChatGPT for learning:

Accessibility: ChatGPT provides immediate access to explanations and solutions, allowing students to learn anytime and anywhere.

Personalized Learning: Students can receive customized explanations tailored to their questions and learning needs.

Interactive Learning: ChatGPT offers an interactive learning experience where students can ask questions and receive instant feedback, promoting active engagement and participation.

Concept Reinforcement: ChatGPT can reinforce students' understanding of concepts by providing multiple explanations and examples in response to their queries.

Disadvantages:

Overdependence: Students may become overly reliant on ChatGPT for answers, potentially hindering their ability to think critically and solve problems independently.

Limited Contextual Understanding: ChatGPT's responses may need more depth and context provided in traditional learning materials, leading to a superficial understanding of concepts.

Misinterpretation: There is a risk of misinterpreting or misunderstanding ChatGPT's responses, especially if the student lacks background knowledge or asks ambiguous questions.

Risk of Errors: ChatGPT may provide incorrect or incomplete information in some cases, leading to misconceptions or errors in learning.

Overall, while ChatGPT can be a valuable tool for learning, students should use it to supplement traditional learning methods and seek guidance from teachers or textbooks to ensure a comprehensive understanding of the subject matter.

Problem

Uncle Jack founded a mathematics department after he got a Ph.D. from one of the top universities in the world 60 years ago. He retired 20 years ago. He is still teaching, doing research, and actively participating in various committees. Now, the department has over 60 faculty members and over 1000 students from over 50 countries. Last month, his university decided to award three graduate students from each of its six colleges the perfect GPA Award, Learning Award, and Research Award.

The Perfect GPA Award is for students with the highest academic standards across their courses, maintaining a perfect Grade Point Average (GPA).

The Learning Award is for students who have demonstrated exceptional engagement with their learning process, perhaps showing notable curiosity, innovation, or contribution to the academic community beyond their grades.

The Research Award is for students who have excelled in research, contributing significantly to their field through publications, conference presentations, and potentially impactful research findings.

The Award and Scholarship Committee at his college, the College of Science, identified three doctoral students: Alex, Nico, and Perry. They are students from Uncle Jack's mathematics department. They all have a 4.0 GPA, have presented at four international conferences, and have published three papers. The Committee consulted with Uncle Jack about distributing the three Awards to those students. Uncle Jack suggested giving the following problem to the students.

Noah plans to buy a beach house. A local bank offers him three mortgage loan (or buying) options—no other fees.

Loan A:

He needs a \$4,000,000 loan. The loan's fixed monthly interest rate is 1% for 30 years.

Loan B:

He needs a \$6,500,000 loan. The loan's fixed monthly interest rate is 0.6% for 30 years.

Loan C:

He has a monthly payment of \$48,000 for 30 years.

Which loan should Noah use?

Alex's Solution

"I used ChatGPT 4 at 8:34 AM on March 25, 2024. It provided the following.

Loan A:

Simplified Monthly Payment: Approximately \$51,111.11

Loan B:

Simplified Monthly Payment: Approximately \$57,055.56

Loan C:

Total Repayment over 30 Years: \$17,280,000

Given that the monthly payment is \$48,000, we can use this to compare without further calculation.

Let's discuss a general problem.

P : mortgage loan principal amount

R : monthly mortgage interest rate

N : number of months to pay off the mortgage loan

M : monthly payment

$B(k)$: current remaining principal loan balance after k months, i.e., the amount still owed on the original amount financed without any interest

Case 1: $R = 0$,

$$B(N) = \frac{P}{N}.$$

Case 2: $N = 0$

$$M = P.$$

Case 3: $N = 1$

$$B(N) = P(1 + R) - M = 0.$$

$$M = P(1 + R).$$

Therefore, Noah should use Loan C."

Nico's Solution

Case 4: $R \neq 0$ and $N > 0$

$$B(1) = P(1 + R) - M$$

$$B(2) = [P(1 + R) - M](1 + R) - M = P(1 + R)^2 - M(1 + R) - M$$

$$B(3) = P(1 + R)^3 - M(1 + R)^2 - M(1 + R) - M$$

$$B(4) = P(1 + R)^4 - M(1 + R)^3 - M(1 + R)^2 - M(1 + R) - M$$

$$B(5) = P(1 + R)^5 - M(1 + R)^4 - M(1 + R)^3 - M(1 + R)^2 - M(1 + R) - M$$

We can use mathematical induction to prove that

$$B(k) = P(1 + R)^k - M[1 + (1 + R) + (1 + R)^2 + (1 + R)^3 + \dots + (1 + R)^{k-1}]$$

$$= P(1 + R)^k - M \frac{1 - (1 + R)^k}{1 - (1 + R)} = P(1 + R)^k + M \frac{1 - (1 + R)^k}{R}$$

$$B(N) = 0$$

$$P(1 + R)^N + M \frac{1 - (1 + R)^N}{R} = 0$$

$$M \frac{1 - (1 + R)^N}{R} = -P(1 + R)^N$$

$$M = \frac{P(1 + R)^N R}{(1 + R)^N - 1}$$

Example

$$N = 1, M = \frac{P(1 + R)R}{R} = P(1 + R)$$

Example

$$N = 2, M = \frac{P(1 + R)^2 R}{(1 + R)^2 - 1} = P \frac{(1 + R)^2}{2 + R}$$

Example, Loan A

$$P = 4000000, R = 0.01, N = 360$$

$$M = \frac{4000000(1 + 0.01)^{360} 0.01}{(1 + 0.01)^{360} - 1} = 41144.5039$$

Example, Loan B

$$P = 6500000, R = 0.006, N = 360$$

$$M = \frac{6500000(1 + 0.006)^{360} 0.006}{(1 + 0.006)^{360} - 1} = 44121.2336$$

Therefore, Noah should use Loan C.

Perry's Solution

Perry's solution contains three parts:

Part 1

"Same as Nico's solution."

Part 2

"I want to introduce the Google Cloud Platform: Colab to you.

Google Compute Engine delivers Virtual Machines (VMs) running in Google's innovative data centers and worldwide fiber network. Computing Engine's VMs boot quickly, come

with persistent disk storage, and deliver high performance. You can set up Colab by following the four steps below.

Step 1: Make sure your computer is connected to the internet.

You can also use your cellphone, Mac, PC, Chromebook, iPad, or Raspberry Pi.

Step 2: Log in with your Gmail account in Google Chrome or another web browser.

Step 3: Open Google Chrome or another web browser you logged into to your Gmail account.

Step 4: In the Search URL bar located at the top of Google Chrome or another web browser, enter the following <http://colab.research.google.com> and press enter.

We could verify the calculations in Part 1 in Colab.”

```
1 P=4000000
2 R=0.01
3 N=360
4 print((P*((1+R)**N)*R)/(((1+R)**N)-1))
5 P=6500000
6 R=0.006
7 N=360
8 print((P*((1+R)**N)*R)/(((1+R)**N)-1))

41144.50387702017
44121.23361538518
```

Part 3

“I appreciate the Award and Scholarship Committee for allowing me to study an exciting problem. I will provide the following table for various cases. I use 10000 as the principal amount of the mortgage loan. If it is not 10000, you can easily adjust it by a factor.”

Table 1

Monthly Interest Rate	Number of Months	Monthly Payment	Total Payments
1%	60	222.44	13346.67
1%	120	143.47	17216.51
1%	180	120.02	21603.03
1%	240	110.11	26426.07
1%	300	105.32	31596.72
1%	360	102.86	37030.05
0.5	60	193.33	11599.68
0.50%	120	111.02	13322.46
0.50%	180	84.38	15189.42
0.50%	240	71.64	17194.35
0.50%	300	64.43	19329.04
0.50%	360	59.96	21583.82
0.30%	60	182.37	10941.93
0.30%	120	99.35	11922.59
0.30%	180	71.98	12956.46
0.30%	240	58.511	14042.68
0.30%	300	50.6	15180.08
0.30%	360	45.47	16367.23
0.25%	60	179.69	10781.21
0.25%	120	96.56	11587.28
0.25%	180	69.06	12430.47
0.25%	240	55.46	13310.34
0.25%	300	47.42	14226.34
0.25%	360	42.16	15177.75

Example of using Table 1:

Assume that Grace has a loan of \$250,000, with a fixed monthly interest rate of 0.25%, for 15 years. Her monthly payment is $2.5 \times 69.06 = 172.65$, and her total payment is 31,076.18.

Award Ceremony

The university president awarded Alex, Nico, and Perry at the Award Ceremony. Each received a certificate and an envelope with a check inside. The amounts on the checks are \$50, \$500, and \$5000. Who got the 50, and who got the 5000?

Problem

The attached paper mentioned that the amounts on the checks were \$50, \$500, and \$5000. How would you distribute the three checks (awards) if you were the judge?

Solution provided by ChatGPT4 (6:33 AM on April 2, 2024)

Based on the information provided in the document and the details of the solutions submitted by Alex, Nico, and Perry, the distribution of the awards would hinge on the depth, creativity, and utility of their respective solutions to the mortgage loan problem set by Uncle Jack. Here is how I would adjudicate the awards:

\$5,000 Award: Perry demonstrated exceptional effort by providing a comprehensive solution to the problem and enhancing the learning experience using Google Cloud Platform: Colab. Perry's solution was detailed, including a table for various cases, which showed a deeper understanding and application of the mathematical concepts involved. This level of detail, innovation, and educational contribution merits the highest award.

\$500 Award: Alex employed ChatGPT to analyze the mortgage loan options and made a direct comparison, suggesting that Loan C was the best option. While this approach was practical and utilized AI tools, it needed more depth and the educational value of Perry's submission. However, Alex's initiative to use AI for problem-solving and correctly identify the most cost-effective loan option warrants recognition with the second-highest award.

\$50 Award: The document needs to provide specific details about Nico's solution, making it difficult to directly evaluate its depth or innovation. Assuming Nico's contribution was accurate but less comprehensive and innovative than Perry's or Alex's, Nico would receive the \$50 award. This recognizes Nico's participation in the challenge and contribution to the problem-solving process.

This distribution of awards reflects a balance between recognizing the complexity and innovation of the solutions and the educational value they provide. Perry's comprehensive and educational approach earns the top prize, Alex's effective use of AI tools secures the second tier, and Nico's participation is acknowledged with the final award, ensuring all contributions are valued.

Remark

ChatGPT4 selected Perry for \$5000. It is correct!
ChatGPT needed help understanding Nico's solution.
ChatGPT selected Alec for \$500. It is incorrect.

Problem

What can we learn from the case studies involving Uncle Jack?

Solution provided by ChatGPT4 (7:40 PM on April 1, 2024)

- (a) **Complex Problem Solving:** Uncle Jack's approach to challenging doctoral students with a complex mathematical problem illustrates the importance of critical thinking and problem-solving skills. It highlights the need to approach problems methodically and use logical reasoning to navigate complexities.
- (b) **Pedagogical Innovation:** By incorporating an unconventional problem into the academic evaluation, Uncle Jack showcases the value of innovative teaching methods. This approach

encourages students to think outside the box and apply their knowledge in novel situations, which is crucial for academic and professional growth.

Our Take on Uncle Jack's Approach

Encouraging Deep Understanding

The nature of the problem given by Uncle Jack emphasizes the importance of a deep understanding of mathematical principles over rote memorization. True mastery of a subject comes from applying foundational concepts to solve new and challenging problems.

- (1) **Integrating Technology in Learning:** The mention of Google Cloud Platform: Colab in Perry's solution, endorsed by Uncle Jack's problem, points to integrating technology in learning and problem-solving. It demonstrates how digital tools and platforms can effectively verify solutions and enhance the learning experience.
- (2) **Valuing Logical Consistency:** The problems and their solutions underscore the importance of logical consistency in mathematical reasoning. They remind us that every step in a solution must be logically sound and based on valid mathematical principles.
- (3) **Learning from Errors:** The document shows that AI, like ChatGPT, can provide incorrect or unreasonable answers to complex problems. This teaches us the value of critically evaluating all solutions, understanding the limitations of tools, and learning from errors to improve future problem-solving efforts.
- (4) **Collaboration and Sharing Knowledge:** Through Perry's willingness to share knowledge about using Google Colab for verification, we learn the importance of cooperation and sharing resources and expertise in academic and professional environments.

Overall, Uncle Jack's case studies offer rich insights into the dynamics of problem-solving, the role of technology in education, and the continuous pursuit of knowledge and understanding in complex domains.

Learning and Knowledge of Agents in Mathematical Problem Solving

The above sections provide a comparative analysis of AI and human approaches to solving mathematical problems. To broaden the scope of this discussion, we will delve deeper into the concepts of learning and knowledge acquisition for agents, which can be either individuals or entities. This expanded perspective will explore how these agents acquire, retain, and apply knowledge in mathematical problem-solving, highlighting their cognitive processes and potential synergies.

Learning in Human Agents

Human learning is a complex, multi-dimensional process that involves acquiring, retaining, and applying knowledge. It can be broken down into several key stages:

- a) **Knowledge Acquisition:** This initial stage involves gathering information through various means such as study, observation, and instruction. This includes understanding fundamental concepts, theories, and problem-solving techniques in mathematics. Human

agents rely on diverse sources, including textbooks, lectures, and interactive learning environments, to build their foundational knowledge (Davies et al., 2021).

- b) **Knowledge Integration:** Individuals synthesize new information with existing knowledge during this stage. Humans utilize cognitive processes such as reasoning, pattern recognition, and logical deduction to integrate new mathematical concepts into their knowledge base. This integration is crucial for developing a comprehensive understanding of mathematical principles and their applications (Laak & Aru, 2024).
- c) **Application and Practice:** Applying acquired knowledge to solve problems reinforces learning. Practice enables the transfer of theoretical knowledge to practical scenarios, enhancing problem-solving skills through repeated application and feedback. This stage is vital for mastering mathematical techniques and strategies (Jia et al., 2024).
- d) **Reflection and Adaptation:** Reflection on problem-solving experiences helps individuals recognize successful strategies and identify areas for improvement. This stage fosters creative and critical thinking, allowing human agents to adapt their knowledge to new, unfamiliar problems. Reflection facilitates learning from both mistakes and successes, refining one's approach to problem-solving (Turing, 1950).

Learning in AI Agents

AI systems, such as ChatGPT, learn through data-driven approaches primarily utilizing machine learning algorithms. The AI learning process can be broadly categorized as follows:

- a) **Data Collection:** AI systems learn from extensive datasets comprising examples of problems and their solutions. In mathematical problem-solving, datasets would include various mathematical problems, their solutions, and the logical steps involved. The quality and diversity of the training data significantly influence the AI's performance (Mohamed et al., 2022).
- b) **Training:** During the training phase, AI models are exposed to the data, learning patterns, and relationships within the dataset. Machine learning algorithms optimize the model parameters to improve performance on given tasks. This phase involves iterative processes where the model adjusts its parameters to minimize errors (Gao et al., 2023).
- c) **Validation and Testing:** AI models are validated using separate datasets to ensure they generalize well to new problems. This phase helps fine-tune the model and identify any overfitting issues. Validation is crucial for ensuring that the AI system can handle a wide range of problems beyond its training data (Raisch & Fomina, 2024).
- d) **Inference and Adaptation:** AI systems can infer solutions to new problems based on learned patterns once trained. However, unlike humans, AI's adaptation is limited to the patterns and rules within its training data. AI lacks the innate ability to reflect and adapt creatively beyond its programmed capabilities (Chesani et al., 2017).

Comparing Human and AI Learning

The learning processes of humans and AI agents highlight distinct strengths and limitations:

- a) **Creativity and Adaptability:** Human agents excel in creative problem-solving and adaptability. They can approach problems from multiple perspectives, utilize analogical reasoning, and adapt strategies based on contextual understanding. AI, while powerful in processing and pattern recognition, often lacks this flexibility and creativity. AI systems follow predefined algorithms and can struggle with problems that require out-of-the-box thinking (Davies et al., 2021).
- b) **Speed and Scale:** AI agents can process and analyze vast amounts of data quickly, identifying patterns and solutions at a scale beyond human capability. This efficiency makes AI valuable in handling large datasets and complex calculations where speed and accuracy are paramount. However, this speed comes at the cost of deep contextual understanding, which humans excel at (Jia et al., 2024).
- c) **Knowledge Retention and Transfer:** Humans retain knowledge through meaningful learning experiences, which can be transferred across various domains. AI's knowledge retention is tied to its training data and model architecture, limiting its ability to generalize knowledge across unrelated domains without additional training. Humans can draw on a wide range of experiences and knowledge to solve new problems, a capability that AI currently lacks (Mohamed et al., 2022).

Implications for Educational Contexts

Incorporating AI tools in education, particularly in teaching mathematical problem-solving, offers significant potential benefits. AI can provide personalized learning experiences, identify areas where students struggle, and offer targeted practice problems to enhance understanding. However, it is crucial to strike a balance, ensuring that AI complements rather than replaces human instruction. Here are some implications and potential strategies for integrating AI into education:

- a) **Personalized Learning:** AI can tailor educational content to meet individual student needs, providing customized practice problems and feedback. This personalization can help address specific weaknesses and reinforce strengths, making learning more efficient and effective (Laak & Aru, 2024).
- b) **Enhanced Engagement:** Interactive AI tools can make learning more engaging by providing instant feedback and creating a more dynamic learning environment. AI can simulate real-world problem-solving scenarios, allowing students to apply their knowledge in practical contexts (Turing, 1950).
- c) **Support for Teachers:** AI can assist teachers by automating routine tasks such as grading and providing insights into student performance. This support allows teachers to focus more on interactive teaching and addressing individual student needs (Raisch & Fomina, 2024).
- d) **Development of Critical Thinking:** While AI can handle routine problem-solving tasks, it is essential to use it to foster critical thinking and creativity in students. Educators should design curricula that challenge students to think deeply and creatively, using AI to support, not replace, these processes (Chesani et al., 2017).

Collaborative Learning Models

A hybrid model where AI and human reasoning converge can leverage the strengths of both agents. For instance, AI can handle routine problem-solving tasks, freeing human instructors to focus on nurturing critical thinking, creativity, and adaptive learning in students. Collaborative learning models can also include AI-assisted tools that provide instant feedback, allowing students to reflect on their problem-solving processes and learn from their mistakes. Such models promote a synergistic approach to learning, combining the efficiency and precision of AI with the depth and adaptability of human cognition (Davies et al., 2021).

Case Studies and Practical Applications

To illustrate the potential of collaborative learning models, consider the following case studies:

- a) **Mathematical Problem-Solving Competitions:** In competitions where students solve complex problems, AI tools can provide real-time feedback on their solutions, helping them refine their approaches. This immediate feedback loop enhances learning and allows students to see the impact of their problem-solving strategies (Jia et al., 2024).
- b) **Classroom Integration:** Teachers can use AI tools to create customized problem sets for students based on their performance. These problem sets can be designed to address specific learning gaps, ensuring that each student receives the support they need to succeed (Mohamed et al., 2022).
- c) **Research Collaboration:** AI can assist researchers in exploring new mathematical theories by identifying patterns and suggesting hypotheses. This collaboration can accelerate the discovery of new knowledge and provide researchers with valuable insights (Gao et al., 2023; Gao et al., 2024).

Remark

In the above section, we reviewed agents' learning and knowledge aspects, enriching the discussion on the interplay between AI and human logic in mathematical problem-solving. Understanding these dimensions highlights the complementary strengths of AI and human agents, paving the way for more effective integration of AI in educational contexts. By fostering a collaborative approach, educators can enhance mathematical problem-solving skills, preparing students for the diverse challenges of the modern world. The future of education lies in harnessing the power of AI to support and enhance human learning, creating a dynamic and innovative learning environment that nurtures critical thinking and creativity.

Conclusion and Future Work

The comparative analysis of AI-generated solutions and human-derived problem-solving strategies in mathematical contexts reveals a complex landscape where AI's computational logic intersects with and sometimes diverges from human cognitive processes. While AI, as exemplified by ChatGPT, demonstrates significant computational prowess, it occasionally needs to catch up in capturing the depth and nuance of human logic, particularly in problems requiring abstract reasoning and creative thought. These findings underscore the importance of a nuanced approach to integrating AI into educational settings. Rather than viewing AI as a replacement for human

reasoning, our analysis advocates for a collaborative model where AI supplements traditional teaching methods, offering students a broader array of problem-solving strategies. This approach leverages AI's computation and data processing strengths and preserves the irreplaceable value of human insight and creativity in education.

Future research should focus on refining AI's reasoning algorithms to better mimic human thought processes, enhancing AI's utility as an educational tool. Additionally, studies should explore pedagogical strategies that effectively integrate AI into curricula, ensuring that AI-assisted learning complements rather than supplants traditional educational methods. In conclusion, the interplay between AI and human logic in mathematical problem-solving presents challenges and opportunities for the field of education. By fostering a synergistic relationship between AI and human reasoning, educators can enhance the teaching and learning of mathematics, equipping students with diverse problem-solving skills suited to the demands of the 21st century.

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